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Complementary Information Measures Based on Three-layer Granular Structures

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ABSTRACT
The measures with three-layer granular structure are the basis of three-layer attribute reduction and machine learning. In the rough set, starts from three-way probabilistic perspectives, the three-way weighted complement-entropies are established by combining different probability products and complementary information, which takes into account the decision table's three-layer granular structures and three-way probabilities. In the neighborhood rough set, this existed neighborhood complement-entropies adopt the complementary mechanism and relate to covering granulation, but don't involve three-way probabilities. In this paper, we mainly presented three-layer algorithms based on three-way neighborhood probabilistic complement-entropies from the Micro-Bottom and Macro-Top. And it also discussed the relationships of complement information measures, namely the three-way neighborhood probabilistic complement-entropies are equivalent to the existing neighborhood complement-entropies, and degenerate into three-way weighted complement-entropies.

KEYWORDS
Neighborhood rough set theory; Three-way probabilities; Complement-entropies; Three-layer granular structures

1 Introduction

In rough set theory [1], the uncertainty measures can describe the system classification ability , which are applied to attribute reductions [2-3], pattern recognition and machine learning [4-6], feature selectionand so on. For the uncertainty measures, there are some good results. For example, Mu et al. [7] improvement establishes double-granule conditional-entropies based on three-level granular structures(i.e., Micro-Bottom, Meso-Middle, Macro-Top). Zhang et al. [8] concretely constructs a D-Tables three-layer granular structures and three-way informational measures via granular computing and Bayes theorem. Wang et al. [9] concretely constructs three-way weighted combination entropies based on the D-Tables three-layer granular structures and Bayes theorem from a new perspective.

In Traditional rough set theory, the requirement of equivalence relation is too restrictive for many practical data sets and applications. To overcome the defects, the notion based on a binary relation of neighborhood rough set was firstly introduced by Yao [10]. On the basis of the distance measures, this measure has been extended in the further studied to obtain more measures in literature [11-12]. By fusing three-way probabilistic granularities, three-way improved neighborhood entropies based on three-level granular structures are proposed: three-way improved neighborhood prior, neighborhood posterior, and neighborhood likelihood entropy [13]. In neighborhood rough set, Zhang et al. [14] proposed classification-level and class-level complement information measures, this is through analytic simulation and information granule substitution, the neighborhood complementary entropy, neighborhood complementary condition entropy and neighborhood complementary mutual information are defined. Although this neighborhood complement-entropies adopt the complementary mechanism and relate to covering granulation, but don't involve three-way probabilities. For this purpose, from three-way probabilistic perspectives, we propose three-way neighborhood probabilistic complement-entropies by combining different probability products and complementary information from the Micro-Bottom and Macro-Top, three-way neighborhood probabilistic complement-entropies include neighbourhood prior, likelihood and posterior complement-entropies, the measures have basic probabilistic semantics and information interpretation.

In this paper, we mainly presented three-layer algorithms based on three-way neighborhood probabilistic complement-entropies from the Micro-Bottom and Macro-Top . And it also discussed the relationships of complement information measures, namely the three-way neighborhood probabilistic complement-entropies are equivalent to the existing neighborhood complement-entropies, and degenerate into three-way weighted complement-entropies.

2 Preliminaries

2.1 Neighborhood Rough Set

In this section, the neighborhood rough set and its existing information measures are viewed by Refs [15-17].

Let $NIS=(U,C,V,f,\delta)$ be a neighborhood information system. Herein, U is a nonempty finite set of objects defined the universe; C is a nonempty finite set of attributes; V is the union of attribute domains such that $V=\bigcup_{c\in C} V_c, f:U\to V_c, c\in C$, $\delta\in[0,1]$ is a neighborhood parameter. More specially, $NDS=(U,C\cup D,V,f,\delta)$ is defined a neighborhood decision system, where$C$ and D are a set of condition and decision attributes, respectively; herein, the equivalent class family determined by decision D is denoted by

$$U/IND=\left\{D_1 \middle| j=1,\cdots,m\right\}. \quad (1)$$

Given a non-empty attribute subset $A=\left\{a_1,\cdots,a_{|A|}\right\}$, which can induce a distance function :

$$d_A(x,y)=\left[\sum_{k=1}^{|A|}\left|v(x,a_k)-v(y,a_k)\right|^p\right]^{\frac{1}{p}} \quad (2)$$

If $p=1$, The distance can be defined as the Manhattan distance, which is adopted in this paper. The distance function satisfies the non-negativity, symmetry and triangular inequality.

Definition 1. The neighborhood relation regarding A is

$$NR_\delta(A)=\left\{(x,y)\in U\times U \middle| d_A(x,y)\le\delta\right\} \quad (3)$$

The neighborhood class of object $x\in U$ regarding A is

$$n_A^\delta(x)=\left\{y\in U \middle| x,y\in NR_\delta(A)\right\}=\left\{y\in U \middle| d_A(x,y)\le\delta\right\} \quad (4)$$

Furthermore, quotient set $U/NR_\delta(A)$ represents the neighborhood knowledge regarding A.

In particular, when $\delta=0$, $NR_\delta(A)$, $n_A^\delta(x)$ and $U/NR_\delta(A)$ would degenerate into the equivalent relation, class and knowledge, respectively. This conclusion shows that neighborhood rough sets expand classical rough sets.

Definition 2[13]. In neighborhood rough set, three-way neighbourhood probabilities

$$P\left(n_A^\delta(x_i)\right)=\frac{\left|n_A^\delta(x_i)\right|}{|U|}, P\left(n_A^\delta(x_i)/D_j\right)=\frac{\left|D_j\cap n_A^\delta(x_i)\right|}{\left|D_j\right|}, P\left(D_j/n_A^\delta(x_i)\right)=\frac{\left|D_j\cap n_A^\delta(x_i)\right|}{\left|n_A^\delta(x_i)\right|}$$

are defined as the neighbourhood prior, posterior, and likelihood probabilities, respectively.

2.2 The Complement Information Measures

In rough set, Tang et al. [18] established the three-way weighted complement-entropies by three-way probabilities. In neighborhood rough system, Zhang et al. [14] constructed neighborhood complement information measures at classification-level and class-level. The definitions referred to in the passage mainly are following:

Definition 3. [18] At the Macro-Top, in the rough set, three-way weighted complement-entropies

$$E_W^D(A)=\sum_{j=1}^{m}\sum_{i=1}^{n} P\left(D_j/[x]_A^i\right)P\left([x]_A^i\right)\left(1-P\left([x]_A^i\right)\right),$$

$$E_W(A/D)=\sum_{j=1}^{m}\sum_{i=1}^{n} P^2\left(D_j\right)P\left([x]_A^i/D_j\right)\left(1-P\left([x]_A^i/D_j\right)\right),$$

$$E_W(D/A)=\sum_{j=1}^{m}\sum_{i=1}^{n} P^2\left([x]_A^i\right)P\left(D_j/[x]_A^i\right)\left(1-P\left(D_j/[x]_A^i\right)\right).$$

are tentatively defined as the prior, posterior, and likelihood weighted complement-entropies, respectively.

Definition 4. [14] At the Macro-Top, in the neighborhood rough set, the neighborhood complement-entropy of A, neighborhood conditional-entropy D about A, neighborhood conditional-entropy A about D, and neighborhood complement-entropy of D are respectively defined as

$$NEC_\delta(A)_D=\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{\left|n_A^\delta(x_i)\cap D_j\right|}{|U|}\frac{\left|n_A^\delta(x_i)^C\right|}{|U|},$$

$$NEC_\delta(A/D)=\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{\left|n_A^\delta(x_i)\cap D_j\right|}{|U|}\frac{\left|\left(n_A^\delta(x_i)\right)^C-\left(D_j\right)^C\right|}{|U|},$$

$$NEC_\delta(D/A)=\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{\left|n_A^\delta(x_i)\cap D_j\right|}{|U|}\frac{\left|\left(D_j\right)^C-\left(n_A^\delta(x_i)\right)^C\right|}{|U|}.$$

where $(*)^C=U-*$ reflects the complementary action.

Via Definition 3 and Definition 4, the neighborhood rough set may degrade into the rough set, on the contrary, the rough set may extend to the neighborhood rough set. For condition knowledge granulation, the condition equivalent class division expand to neighborhood covering, equivalent class expand to neighborhood granule.

3 Three-way Neighbourhood Probabilistic Complement-entropies Based on Three-layer Granular Structures

In this section, three-way neighbourhood probabilistic complement-entropies are tactfully induced by Bayes' probability formula, which are hierarchically constructed by using three-layer granular structures.

3.1 Three-layer Algorithms Based on Three-way Neighborhood Probabilistic Complement-entropies

In this subsection, the definitions of three-way NPCEs based on three-way neighbourhood probabilities are given from Micro-Bottom to Macro-Top in the following.

Definition 5. At the Meso-Middle, three-way neighbourhood probabilistic complement-entropies

$$NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j}=P\left(D_j/n_A^{\delta}(x_i)\right)P\left(n_A^{\delta}(x_i)\right)\left(1-P\left(n_A^{\delta}(x_i)\right)\right),$$

$$NHC_{lk}^{\delta}\left(D_j/n_A^{\delta}(x_i)\right)=P^2\left(n_A^{\delta}(x_i)\right)P\left(D_j/n_A^{\delta}(x_i)\right)\left[1-P\left(D_j/n_A^{\delta}(x_i)\right)\right],$$

$$NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right)=P^2\left(D_j\right)P\left(n_A^{\delta}(x_i)/D_j\right)\left(1-P\left(n_A^{\delta}(x_i)/D_j\right)\right).$$

are defined as the neighbourhood prior, likelihood and posterior probabilistic complement-entropies, respectively.

Definition 6. At the Meso-Middle, three-way neighbourhood probabilistic complement-entropies

$$NHC_{pr}^{\delta}(A)_{D_j}=\sum_{i=1}^{n}NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j},$$

$$NHC_{lk}^{\delta}\left(D_j/A\right)=\sum_{i=1}^{n}NHC_{lk}^{\delta}\left(D_j/n_A^{\delta}(x_i)\right),$$

$$NHC_{po}^{\delta}\left(A/D_j\right)=\sum_{i=1}^{n}NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right).$$

are defined as the neighbourhood prior, likelihood and posterior probabilistic complement-entropies, respectively.

Definition 7. At the Macro-Top, the three-way neighbourhood probabilistic complement-entropies

$$NHC_{pr}^{\delta}(A)_{D}=\sum_{j=1}^{m}NHC_{pr}^{\delta}(A)_{D_j},$$

$$NHC_{lk}^{\delta}(D/A)=\sum_{j=1}^{m}NHC_{lk}^{\delta}\left(D_j/A\right),$$

$$NHC_{po}^{\delta}(A/D)=\sum_{j=1}^{m}NHC_{po}^{\delta}\left(A/D_j\right).$$

are defined as the neighbourhood prior, likelihood and posterior probabilistic complement-entropies, respectively.

To calculate three-way NPCEs from Micro-Bottom to Macro-Top. Algorithm 1 uses three " for" loop statements to complete the integration of granules. It include: For three-way NPCEs in Micro-Bottom, their calculations run from 6 to 8 steps; In Meso-Middle, from step 3 to 10, by integrating *i*, according to Definition 6, we obtain the values of three-way NPCEs; Aiming at Macro-Top, in the j loop interior with Steps 1-12, Step 1 assigns an initial value of 0, we utilize the sum renewal for *j* (Definition 7) to gradually gain the top measures.

Algorithm 1 Algorithm of the three-way NPCEs from Micro-Bottom to Macro-Top

Require: Granules $n_A^{\delta}(x_i)\in U/NR_{\delta}(A)$ $(i=1,\cdots,n)$ and class$D_j\in U/IND(D)$;

Ensure: Three-way neighbourhood probabilistic complement-entropies:

1:Assign top value 0: $NHC_{pr}^{\delta}(A)_{D}=0, NHC_{lk}^{\delta}(D/A)=0, NHC_{po}^{\delta}(A/D)=0.$

2: for $j\in\{1,\cdots,m\}$ do

3: Assign middle value 0:$NHC_{pr}^{\delta}(A)_{D_j}=0, NHC_{lk}^{\delta}\left(D_j/A\right)=0, NHC_{po}^{\delta}\left(A/D_j\right)=0.$

4: for $i\in\{1,\cdots,n\}$ do

5: Assign middle value 0: $NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j}=0, NHC_{lk}^{\delta}\left(D_j/n_A^{\delta}(x_i)\right)=0, NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right)=0.$

6: for $x\in U$ do

7: By Definition 5, make a product calculation:

$$NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j}\leftarrow NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j}+P\left(D_j/n_A^{\delta}(x_i)\right)P\left(n_A^{\delta}(x_i)\right)\left(1-P\left(n_A^{\delta}(x_i)\right)\right),$$

$$NHC_{lk}^{\delta}\left(D_j/n_A^{\delta}(x_i)\right)\leftarrow NHC\left(D_j/n_A^{\delta}(x_i)\right)+P^2\left(n_A^{\delta}(x_i)\right)P\left(D_j/n_A^{\delta}(x_i)\right)\left[1-P\left(D_j/n_A^{\delta}(x_i)\right)\right],$$

$$NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right)\leftarrow NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right)+P^2\left(D_j\right)P\left(n_A^{\delta}(x_i)/D_j\right)\left(1-P\left(n_A^{\delta}(x_i)/D_j\right)\right),$$

8: end for

9: By Definition 6, make a summation renewal:

$$NHC_{pr}^{\delta}(A)_{D_j} \leftarrow NHC_{pr}^{\delta}(A)_{D_j} + NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j},$$

$$NHC_{lk}^{\delta}\left(D_j/A\right) \leftarrow NHC_{lk}^{\delta}\left(D_j/A\right) + NHC_{lk}^{\delta}\left(D_j/n_A^{\delta}(x_i)\right),$$

$$NHC_{po}^{\delta}\left(A/D_j\right) \leftarrow NHC_{po}^{\delta}\left(A/D_j\right) + NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right).$$

10: end for

11: By Definition 7, make a summation renewal:

$$NHC_{pr}^{\delta}(A)_D \leftarrow NHC_{pr}^{\delta}(A)_D + NHC_{pr}^{\delta}(A)_{D_j},$$

$$NHC_{lk}^{\delta}(D/A) \leftarrow NHC_{lk}^{\delta}(D/A) + NHC_{lk}^{\delta}\left(D_j/A\right),$$

$$NHC_{po}^{\delta}(A/D) \leftarrow NHC_{po}^{\delta}(A/D) + NHC_{po}^{\delta}\left(A/D_j\right).$$

12: end for

13: return $NHC_{pr}^{\delta}(A)_D, NHC_{lk}^{\delta}(D/A), NHC_{po}^{\delta}(A/D)$.

3.2 The Relationships of Complement Information Measures

Thus far, three-way NPCEs have been constructed, they have good mechanism, hierarchy, systematicness, monotonicity/non-monotonicity. Herein, we mainly analyze the relationships of three-way neighbourhood probabilistic complement-entropies (three-way NPCEs), the existing neighbourhood complement-entropies(NCEs) and three-way weighted complement-entropies (three-way WCEs).

Theorem 1. At the Micro-Bottom, the three-way NPCEs and NCEs are equivalent. Namely,

$$NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j} = NEC_{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j},$$

$$NHC_{lk}^{\delta}\left(D_j/n_A^{\delta}(x_i)\right) = NEC_{\delta}\left(D_j/n_A^{\delta}(x_i)\right),$$

$$NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right) = NEC_{\delta}\left(n_A^{\delta}(x_i)/D_j\right).$$

Proof.

$$NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j} = P\left(D_j/n_A^{\delta}(x_i)\right)P\left(n_A^{\delta}(x_i)\right)\left(1-P\left(n_A^{\delta}(x_i)\right)\right)$$

$$= \frac{\left|D_j \cap n_A^{\delta}(x_i)\right|}{\left|n_A^{\delta}(x_i)\right|}\frac{\left|n_A^{\delta}(x_i)\right|}{|U|}\frac{|U|-\left|n_A^{\delta}(x_i)\right|}{|U|}$$

$$= \frac{\left|D_j \cap n_A^{\delta}(x_i)\right|}{|U|}\frac{|U|-\left|n_A^{\delta}(x_i)\right|}{|U|} = NEC_{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j}.$$

$$NHC_{lk}^{\delta}\left(D_j/n_A^{\delta}(x_i)\right) = P^2\left(n_A^{\delta}(x_i)\right)P\left(D_j/n_A^{\delta}(x_i)\right)\left[1-P\left(D_j/n_A^{\delta}(x_i)\right)\right]$$

$$= \frac{\left|n_A^{\delta}(x_i)\right|^2}{|U|^2}\frac{\left|D_j \cap n_A^{\delta}(x_i)\right|}{\left|n_A^{\delta}(x_i)\right|}\frac{\left|n_A^{\delta}(x_i)\right|-\left|D_j \cap n_A^{\delta}(x_i)\right|}{\left|n_A^{\delta}(x_i)\right|}$$

$$= \frac{\left|D_j \cap n_A^{\delta}(x_i)\right|}{|U|}\frac{\left|n_A^{\delta}(x_i)\right|-\left|D_j \cap n_A^{\delta}(x_i)\right|}{|U|} = \frac{\left|D_j \cap n_A^{\delta}(x_i)\right|}{|U|}\frac{\left|n_A^{\delta}(x_i)-D_j\right|}{|U|}$$

$$= NEC_{\delta}\left(D_j/n_A^{\delta}(x_i)\right).$$

$$NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right) = P^2\left(D_j\right)P\left(n_A^{\delta}(x_i)/D_j\right)\left(1-P\left(n_A^{\delta}(x_i)/D_j\right)\right)$$

$$= \frac{\left|D_j\right|^2}{|U|^2}\frac{\left|D_j \cap n_A^{\delta}(x_i)\right|}{\left|D_j\right|}\frac{\left|D_j\right|-\left|D_j \cap n_A^{\delta}(x_i)\right|}{\left|D_j\right|} = \frac{\left|D_j \cap n_A^{\delta}(x_i)\right|}{|U|}\frac{\left|D_j\right|-\left|D_j \cap n_A^{\delta}(x_i)\right|}{|U|} = NEC_{\delta}\left(n_A^{\delta}(x_i)/D_j\right).$$

Theorem 2. At the Meso-Middle, the three-way NPCEs and NCEs are equivalent. Namely,

$$NHC_{pr}^{\delta}(A)_{D_j} = NEC_{\delta}(A)_{D_j},$$

$$NHC_{lk}^{\delta}\left(D_j/A\right) = NEC_{\delta}\left(D_j/A\right),$$

$$NHC_{po}^{\delta}\left(A/D_j\right) = NEC_{\delta}\left(A/D_j\right).$$

Theorem 3. At the Macro-Top, the three-way NPCEs and NCEs are equivalent. Namely,

$$NHC_{pr}^{\delta}(A)_D = NEC_{\delta}(A)_D,$$

$$NHC_{lk}^{\delta}(D/A) = NEC_{\delta}(D/A),$$

$$NHC_{po}^{\delta}(A/D) = NEC_{\delta}(A/D).$$

If δ = 0, then the three-way neighbourhood probabilistic complement-entropies will degrade into three-way weighted complement-entropies, so we have three Propositions in the next.

Proposition 1. At the Micro-Bottom, the three-way NPCEs degrade into three-way WCEs, that is

$$NHC_{pr}^{\delta}\left(n_A^{\delta}(x_i)\right)_{D_j} = E_W^{D_j}\left([x]_A^i\right),$$

$$NHC_{lk}^{\delta}\left(D_j/n_A^{\delta}(x_i)\right) = E_W\left(D_j/[x_i]_A^i\right),$$

$$NHC_{po}^{\delta}\left(n_A^{\delta}(x_i)/D_j\right) = E_W\left([x_i]_A^i/D_j\right).$$

Proposition 2 At the Meso-Middle, the three-way NPCEs degrade into three-way WCEs, that is

$$NHC_{pr}^{\delta}(A)_{D_j} = E_W^{D_j}(A),$$

$$NHC_{lk}^{\delta}\left(D_j/A\right) = E_W\left(D_j/A\right),$$

$$NHC_{po}^{\delta}\left(A/D_j\right) = E_W\left(A/D_j\right).$$

Proposition 3. At the Macro-Top, the three-way NPCEs degrade into three-way WCEs, that is

$$NHC_{pr}^{\delta}(A)_D = E_W^D(A),$$

$$NHC_{lk}^{\delta}(D/A) = E_W(D/A),$$

$$NHC_{po}^{\delta}(A/D) = E_W(A/D).$$

Herein, For three-way NPCEs, the existing NCEs and three-way WCEs, their equivalency and degeneracy are shown in Figure 1.

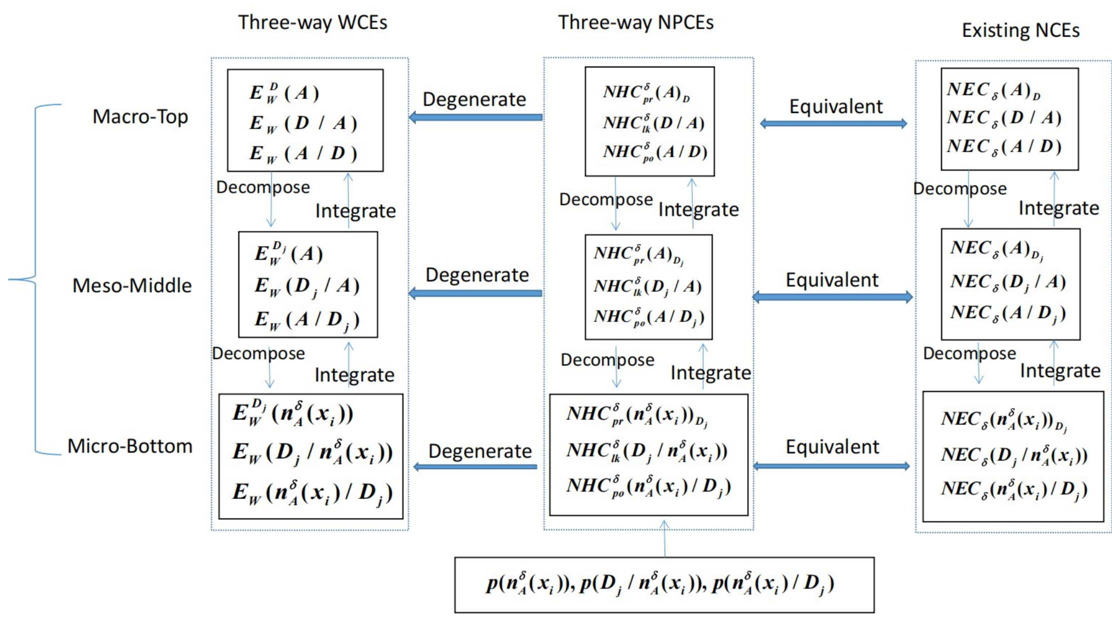

Figure 1 The characteristic summary and mutual relationships of complement information measures

From Figure 1, we know the three-way probabilities provide an effective method to construct three-way NPCEs. At the Micro-Bottom, through the transformation of relations, we gain the equivalence property between the three-way NPCEs and NCEs . If δ = 0, the three-way NPCEs degrade into three-way WCEs.

In summary, the three-way neighbourhood probabilistic complement-entropies and neighbourhood complement-entropies have different forms, but the two systems acquire same equivalent relationships by granular hierarchy. Hence, the neighbourhood probabilistic complement-entropy system deepen the structure and mechanism of neighbourhood complement-entropy, and expand three-way weighted complement-entropy.

4 Conclusion

In this paper, from three-way probabilistic perspectives, we propose three-way neighborhood probabilistic complement-entropies by combining different probability products and complementary information from the Micro-

Bottom and Macro-Top. We mainly presented three-layer algorithms based on three-way neighborhood probabilistic complement-entropies from the Micro-Bottom and Macro-Top. And it also discussed the relationships of complement information measures, namely the three-way neighborhood probabilistic complement-entropies are equivalent to the existing neighborhood complement-entropies, and degenerate into three-way weighted complement-entropies.

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